Application of n-fold Interval Spherical Uncertanity sets over Implicatives Subalgebra.

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Abstract: In this paper,we introduce the concepts of n-fold interval spherial uncertanity ideals, and investigate several properties, we provide condition for an DEL n-fold spherical uncertanity set to be an implicative BDEL-spherical uncertanity ideal and discoverd relations between them.Characterzation of positive implicative CDEL-spherical uncertanity ideal are displayed.
Keywords: BCK/BCK- algebra,n-fold implicative algebra,interval,uncertanity set,spherical uncertanity set,DEL-spherical uncertanity ideal,BDEL-spherical uncertanity.

1 Introduction

After the presentation of ordinary fuzzy sets by Zadeh [12], they have been very popular in almost all branches of science. Various researchers have developed several extensions of ordinary fuzzy sets. In recent years, several researchers have utilized these extensions in the solution of multi-criteria decision making problems. A classification of some recent representative publications with respect to the types of fuzzy extensions is as follows: Type-2 fuzzy sets (T2FS): The concept of a type-2 fuzzy set was introduced by Zadeh [12] as an extension of the concept of an ordinary fuzzy set called a type-1 fuzzy set. Such sets are fuzzy sets whose membership grades themselves are type-1 fuzzy sets; they are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set. Intuitionistic fuzzy sets (IFS): Intuitionistic fuzzy sets introduced by Atanassov [1] enable defining both the membership and non-membership degrees of an element in a fuzzy set. Hesitant fuzzy sets (HFS): Hesitant fuzzy sets can be used as a functional tool allowing many potential degrees of membership of an element to a set. These fuzzy sets force the membership degree of an element to be possible values between zero and one [6]. Pythagorean fuzzy sets (PFS): Atanassov's intuitionistic fuzzy sets of second type (IFS2) have been renamed by Yager [10] as Pythagorean fuzzy sets (PFS). Hence PFS and IFS2 mean the same fuzzy sets thereafter. Atanassov's intuitionistic fuzzy sets of second type (IFS2) or Yager's Pythagorean fuzzy sets are characterized by a membership degree and a nonmembership degree satisfying the condition that the square sum of its membership degree and nonmembership degree is equal to or less than one, which is a generalization of Intuitionistic Fuzzy Sets (IFS). Neutrosophic sets (NS): Smarandache [7] developed neutrosophic logic and neutrosophic sets (NSs) as an extension of intuitionistic fuzzy sets. The neutrosophic set is defined as the set where each element of the universe has a degree of truthiness, indeterminacy and falsity. KutluGundog?du and Kahraman ([4], [5], [6]) have recently introduced the spherical fuzzy sets (SFS). These sets are based on the fact that the hesitancy of a decision maker can be defined independently from membership and non-membership degrees, satisfying the following some certain condition where each valuesare the degrees of membership, non-membership, and hesitancy of u to $A \sim$ for each u, respectively. On the surface of the sphere, the idea behind SFS is to let decision makers to generalize other extensions of fuzzy sets by defining a membership function on a spherical surface and independently assign the parameters of that membership function with a larger domain. SFS are a synthesis of PFS and NS.In this paper,we introduce the concepts of n-fold interval spherial uncertanity ideals, and investigate several properties, we provide condition for an DEL n-fold spherical uncertanity set to be an implicative BDEL-spherical uncertanity ideal and discoverd relations between them.Characterzation of positive implicative CDEL-spherical uncertanity ideal are displayed.

2 Preliminaries and Basic Concepts

By a BCI-algebra,we mean set X with a binary operation $*$and a special elements 0 that satisfies the following conditions:

- $(BCI-1) : ((x * y) * (x * z)) * (z * y) = 0$
- $(BCI-2) : ((x * (x * y) * y) = 0$
- $(BCI-3) : ((x * x) = 0$

- $(BCI-4):((x*y)=0,(y*x))=0 \Rightarrow x=y, \forall\ x,y,z \in X$. If a BCI-algebra X satisfies the following identity:
- $(BCI-5):((0*x)=0\ \forall\ x \in X$,then X is called a BCK-Algebra.
- $(BCI-6)$: (T,S)-Normed interval spherical uncertanity set over implicative subalgebra.

Every BCK/BCI-algebra$X$ satisfies the following conditions:

- $x*0=x \longrightarrow (1)$
- $x \leq y \Rightarrow x*z \leq y*z, z*y \leq z*x \longrightarrow (2)$
- $(x*y)*z=(x*z)*y \longrightarrow (3)$
- $(x*z)*(y*z) \leq x*y, \forall\ x,y,z \in X \longrightarrow (4)$

where $x \leq y$ if and only if $x*y=0$
A non empty subset S of a BCK/BCI-algebra χ is called a subalgebra of x if $x,y \in S\ \forall\ x,y \in S$. A subset I of a BCK/BCI-algebraχ is called an ideal of χ if it satisfies:

- $0 \in I \longrightarrow (5)$
- $(x*y) \in I \Rightarrow x \in I$, $\forall x,y \in I \longrightarrow (6)$

A subset I of a BCK algebra χ is called (see J.Meng and Y.B.Jun)a n-fold positive implicative ideal of χ if satisfies (5) and

- $(x*y^{(n+1)})*Z \in I\ y*z \in I \Rightarrow x*z \in I \longrightarrow (7)\ \forall x,y,z \in X$

Note from [J.Meng and Y.B.Jun] that a subset I of a BCK-algebra χ is a n-fold positive implicative ideal of χ if and ony if it is on ideal of χ which is satisfeis the condition

- $(x*y^{(n+1)})*y \in I \Rightarrow x*y \in I\ \forall\ x,y \in X \longrightarrow$(8)

By an interval number we mean a closed subinterval $C = [a^+, a^-]$ of I, where $0 \leq a^+ \leq a- \leq 1$. Denote by [I]the set of all interveal numbers.Let us define what is know as refind minimum (briefly,rmin) and refined maximum(briefly,rmax)of two elements in [I]. we also define the symbals "$\leq$"," $\geq$ "," $=$ " in case of two elements in [I].

Consider two interval numbers $\tilde{a_1} = [{a_1}^+, {a_1}^-]$ and $\tilde{a_2} = [{a_2}^+, {a_2}^-]$, then
rmin $\{\tilde{a_1}, \tilde{a_2}\}$= [min $\{{a_1}^-, {a_1}^-\}$, min $\{{a_1}^+, {a_1}^+\}$]
rmax $\{\tilde{a_1}, \tilde{a_2}\}$= [max $\{{a_1}^-, {a_1}^-\}$, max $\{{a_1}^+, {a_1}^+\}$]
$\tilde{a_1} \geq \tilde{a_2} \Leftrightarrow {a_1}^- \geq {a_2}^-, {a_1}^+ \geq {a_2}^+$ and similarly we may have $\tilde{a_1} \leq \tilde{a_2}$ and $\tilde{a_1} = \tilde{a_2}$.To say $\tilde{a_1} > \tilde{a_2}$ (respetively $\tilde{a_1} < \tilde{a_2}$) we mean $\tilde{a_1} \geq \tilde{a_2}$ and $\tilde{a_1} \neq \tilde{a_2}$ ($\tilde{a_1} < \tilde{a_2}$ and $\tilde{a_1} \neq \tilde{a_2}$)
Let $\tilde{a}_i \in [I]$ where $i \in \Lambda$.we define

$$r \inf_{i \in \Lambda} \tilde{a}_i = \left[\inf_{i \in \Lambda} a_i^-, \ \inf_{i \in \Lambda} a_i^+ \right] \text{ and } r \sup_{i \in \Lambda} \tilde{a}_i = \left[\sup_{i \in \Lambda} a_i^-, \ \sup_{i \in \Lambda} a_i^+ \right]$$

Let x be a non-empty set. A function $A : X \to [I]$ is called on interval-valued fuzzy set (briefly, an IVF set) in X. Let $[I]^X$ stand for the set of all IVF sets in X. For every $A \in [I]^X$ and $x \in X$, $A(x) = [A^-(x), A^+(x)]$ is called the degree of membership of on element x to A, where $A^- : X \to I$ and $A^+ : X \to I$ are fuzzy sets in X which is called lower set and upper fuzzy set in X, respectively.For simplicity ,we have denote $A = [A^-, A^+]$.

2.1 Definition[R.R.Yagar 2013]

A pythagorean fuzzy set (PyFs) on X consists of membership and non-membership function defined as $M = \{\langle x, D(x), E(x) \rangle / x \in X\}$ such that $D, E : X \to [0, 1]$ with a condition that $0 \leq D^2(x) + E^2(x) \leq 1 \ \forall x \in X$. Further, the degree of refusal of x in M is $r(x) = \sqrt{1 - (D^2(x) + E^2(x))}$ and the pair (D, E) is regorded as a pythogorean fuzzy numbers (PyFN).

2.2 Definition[Wong.B.C 2013]

A picture fuzzy set (PFs) on X consists of membership,abstinence and non-membership function defined as $M = \{\langle x, D(x), E(x), L(x) \rangle /x \in X\}$such that $D, E, L : X \to [0, 1]$ with a condition that $0 \leq D(x)+E(x)+L(x) \leq 1 \ \forall x \in X$. Further, the degree of refusal of x in M is $r(x) = 1 - (D(x) + E(x) + L(x))$ and (D,E,L) is represented as picture fuzzy number(PFN).

2.3 Definition[Mohamood.T 2018]

A spherical fuzzy set (SFs) on X consists of membership interval valued set (IVF-set) and non-membership functions defined as
$A = \{\left\langle x, D_A(x), \tilde{E}_A(x), L_A(x) \right\rangle /x \in X\}$such that $D_A, \tilde{E}_A, L_A : X \to [0, 1]$ with a condition that $0 \leq D_A^2(x) + E_A^2(x) + L_A^2(x) \leq 1 \ \ \forall x \in X$. Further, the degree of refusal of x in A is $r(x) = \sqrt{1 - (D_A^2(x) + \tilde{E}_A^2(x) + L_A^2(x))}$ and (D_A, E_A, L_A) is represented as spherical fuzzy number(SFN). For the sake of simplicty,we shall use the symbol $A = (D_A, \tilde{E}_A, L_A)$ for the DEL-spherical fuzzy set $A = \{\left\langle x, D_A(x), \tilde{E}_A(x), L_A(x) \right\rangle /x \in X\}$.

2.4 Definition

Let X be a BCK/BCI-algebra. An DEL-spherical fuzzy set $A = (D_A, \tilde{B}_A, L_A)$ in X is called n-fold DEL-spherical fuzzy ideal of X if it satisfies:

- $D_A(x) + E_A^-(x) \leq 1 \ \ E_A^+(x) + L_A(x) \leq 1 \longrightarrow (9) \ \forall x \in X$
- $D_A(0) \geq D_A(x), \ E_A^-(0) \leq E_A^+(x), \ E_A^+(0) \geq E_A^+(x), \ \ L_A(0) \leq L_A(x) \longrightarrow$ (10) $\quad \forall x \in X$
- $D_A(x) \geq min \ \{D_A(x * y^{(n+1)}), D_A(y)\}$
- $E_A^-(x) \leq max \ \{E_A^-(x * y^{(n+1)}), E_A^-(y)\}$
- $E_A^+(x) \geq min \ \{E_A^+(x * y^{(n+1)}), E_A^+(y)\}$
- $L_A(x) \leq max \ \{L_A(x * y^{(n+1)}), L_A(y)\} \ \forall x, y \in X \longrightarrow (11)$.

for the sake of simplicity,we shall symbol $A = \left\langle D_A, \tilde{B}_A, L_A \right\rangle$ for the BDEL-spherical fuzzy set $A = \{\left\langle x, D_A(x), \tilde{E}_A(x), L_A(x) \right\rangle /x \in X\}$.

3 Implicative structures of BDEL-spherical fuzzy ideals

In what follows, let X denote a BCK-algebra unless otherwise specified.

3.1 Definition

An BDEL-spherical fuzzy set A in X is called a n-fold positive fuzzy implicative BDEL-spherical fuzzy ideal of if satisfies (9)(10) and

- $D_A(x * z) \geq min\ \{D_A((x * y^{(n+1)}) * z), D_A(y * z)\}$
- $E_A^-(x * z) \leq max\ \{E_A^-((x * y^{(n+1)}) * z), E_A^-(y * z)\}$
- $E_A^+(x * z) \geq min\ \{E_A^+((x * y^{(n+1)}) * z), E_A^+(y * z)\}$
- $L_A(x * z) \leq max\ \{L_A((x * y^{(n+1)}) * z), L_A(y * z)\}\ \forall x, y \in X \longrightarrow (12)$.

3.2 Example

Consider a BCK-algebra X=$\{0, 1, 2, 3, 4\}$ with binary operation $*$ which is given in Table -1.
Let A be an DEL-spherical fuzzy set in X defined by Table-2

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	0	0
2	2	2	0	0	2
3	3	3	3	0	3
4	4	4	4	4	0

Table-1

and

X	$D_A(x)$	$E_A(x)$	$L_A(x)$
0	0.86	$[0.02, 0.07]$	0.34
1	0.77	$[0.04, 0.09]$	0.42
2	0.62	$[0.01, 0.09]$	0.57
3	0.53	$[0.05, 0.07]$	0.63
4	0.42	$[0.04, 0.09]$	0.77

Table-2 DEL-spherical fuzzy set A it is routine to verify that A is a n-fold positive implicater BDEL-spherical fuzzy ideal of X.

3.3 Theorem

Every n-fold positive implicative BDEL-spherical fuzzy ideal is a BDEL- spherical fuzzy ideal.

Proof:

The condition (11) is induced by taking Z=0 in (12). Hence every n-fold positive implicative BDEL-spherical fuzzy ideal is a BDEL-spherical fuzzy idea. The convence of theorem is not true as seen in the following example.

3.4 Example

Consider a BCK-algebra X={0, 1, 2, 3,} with binary operation $*$ which is given in Table -3.Cayley take for the binary operation $*$

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	1	0	2
3	3	3	3	0

Let A be an DEL-spherical fuzzy set in X defined by Table-4

X	$D_A(x)$	$\overline{E}_A(x)$	$L_A(x)$
0	0.8	[0.06, 0.09]	0.2
1	0.7	[0.05, 0.07]	0.9
2	0.7	[0.04, 0.08]	0.9
3	0.2	[0.02, 0.03]	0.7

If is routine to verifes that A is a BDEL-spherical fuzzy ideal of X.But it is not a n-fold positive implicative DEL-spherical fuzzy ideal of X since
$D_A(2 * 3)$=0.7¡0.8= $min\{D_A(0.8, 0.8)\}$

3.5 Lemma

Every n-fold BDEL-spherical fuzzy ideal A of X satisfies the following assertion:
$x \leq y \Rightarrow \begin{pmatrix} D_A(x) \geq D_A(y) & E_A^-(x) \leq E_A^-(y) \\ E_A^+(0) \geq E_A^+(x) & L_A(x) \leq L_A(y) \end{pmatrix} \forall\ x, y \in X \longrightarrow (13)$

Proof:

Assume that $x \leq y \ \forall \ x, y \in X$.Then$x * y = 0$,and so on

$D_A(x) \geq min\ \{D_A(x * y^{(n+1)}), D_A(y)\}$= $\min\{D_A(0), D_A(y)\}$=$D_A(y)$.

$E_A^-(x) \leq max\ \{E_A^-(x * y^{(n+1)}), E_A^-(y)\}$= $\max\{E_A^-(0), E_A^-(y)\}$=$E_A^-(y)$.

$E_A^+(x) \geq min\ \{E_A^+(x * y^{(n+1)}), E_A^+(y)\}$= $\min\{E_A^+(0), E_A^+(y)\}$=$E_A^+(y)$.

and

$L_A(x) \leq max\ \{L_A(x * y^{(n+1)}), L_A(y)\}$= $\max\{L_A(0), L_A(y)\}$=$L_A(y)$.

Hence the proof.

3.6 Note

In this section we prouide condition for a n-fold BDELspherical fuzzy ideal to be an-fold positive implicative BDEL-spherical ideal.

3.7 Theorem

An DEL-sphericaal fuzzy set A in X is a n-fold positive implicative BDEL-spherical ideal of X if and only if it is BDEL-spherical fuzzy ideal of X and satisfies the following condition: $D_A(x * y) \geq \{D_A((x * y^{(n+1)}) * y)\}$,
$E_A^-(x * y) \leq \{E_A^-((x * y^{(n+1)}) * y)\}$ $E_A^+(x * y) \geq \{E_A^+((x * y^{(n+1)}) * y)$
$L_A(x * y) \leq \{L_A((x * y^{(n+1)}) * y)\}$ $\forall x, y \in X \longrightarrow (14)$.

Proof:

Assume that A is n-fold positive implicative DEL-spherical fuzzy ideal of X. If Z is replaced by y in Equation (12),then

$$\begin{aligned}
D_A(x * y) &\geq min\{D_A((x * y^{(n+1)}) * y), D_A(y * y)\} \\
&= min\{D_A((x * y^{(n+1)}) * y), D_A(0)\} \\
&= D_A((x * y^{(n+1)}) * y)\} \\
E_A^-(x * y) &\leq max\{E_A^-((x * y^{(n+1)}) * y), E_A^-(y * y)\} \\
&= max\{E_A^-((x * y^{(n+1)}) * y), E_A^-(0)\} \\
&= E_A^-((x * y^{(n+1)}) * y)
\end{aligned}$$

$$\begin{aligned}
E_A^+(x*y) &\geq min\{E_A^+((x*y^{(n+1)})*y), E_A^+(y*y)\}\\
&= min\{E_A^+((x*y^{(n+1)})*y), E_A^+(0)\}\\
&= E_A^+((x*y^{(n+1)})*y)\\
L_A(x*y) &\leq max\{L_A((x*y^{(n+1)})*y), L_A(y*y)\}\\
&= max\{L_A((x*y^{(n+1)})*y), L_A(0)\}\\
&= L_A((x*y^{(n+1)})*y)\forall x, y \in X.
\end{aligned}$$

Conversely,let A be an DEL-spherical fuzzy ideal of X satisfuing the condition (14) since

$$\begin{aligned}
((x*z)*z)*(y*z) &\leq ((x*z)*y)\\
&= (x*y)*z\forall x, y, z \in X.
\end{aligned}$$

It follows from lemma that

$$\begin{aligned}
D_A((x*y^{n+1})*z) &\leq D_A((x*z)*z)*(y*z)\\
E^-{}_A((x*y^{n+1})*z) &\geq E^-(((x*z)*z)*(y*z))\\
E^+{}_A((x*y^{n+1})*z) &\leq E^+(((x*z)*z)*(y*z))\\
L_A((x*y^{n+1})*z) &\geq L_A(((x*z)*z)*(y*z))\forall x, y, z \in X \longrightarrow (15).
\end{aligned}$$

Using equation (14),(11)and (15) we have

$$\begin{aligned}
D_A(x*z) &\geq D_A((x*z)*z)\\
&\geq min\{D_A((x*z)*z)*(y*z), D_A(y*z)\}\\
&\geq min\{D_A((x*y^{(n+1)})*z), D_A(y*z)\}\\
E_A^-(x*z) &\leq E_A^-((x*z)*z)\\
&\leq max\{E_A^-((x*z)*z)*(y*z), E_A^-(y*z)\}\\
&\leq max\{E_A^-((x*y^{(n+1)})*z), E_A^-(y*z)\}\\
E_A^+(x*z) \geq E_A^+((x*z)*z) &\geq min\{E_A^+((x*z)*z)*(y*z), E_A^+(y*z)\}\\
&\geq min\{E_A^+((x*y^{(n+1)})*z), E_A^+(y*z)\} \text{ and}\\
L_A(x*z) &\leq L_A((x*z)*z)\\
&\leq max\{L_A((x*z)*z)*(y*z), L_A(y*z)\}\\
&\leq max\{L_A((x*y^{(n+1)})*z), L_A(y*z)\}\forall x, y, z \in X.
\end{aligned}$$

Therefore A is a n-fold positive implicative BDEL-spherical fuzzy ideal of X.

3.8 Definition

Given an DEL-spherical set A in X, we consider the following upper and lower bound sets. $U(D_A;t)$=$\{x \in X/D_A(x) \geq t\}$
$L(E_A^-;\alpha^-)$=$\{x \in X/E_A^-(x) \leq \alpha^-\}$
$U(E_A^+;\alpha^+)$=$\{x \in X/E_A^+(x) \geq \alpha^+\}$
$L(L_A;S)$=$\{x \in X/L_A(x) \leq S\}$ $\forall\ t, s, \alpha^-\alpha^+ \in [0,1]$.

3.9 Lemma

An DEL-spherical fuzzy set A in X is a BDEL-spherical fuzzy ideal of X if and only if the non-empty sets $U(D_A;t)$,$L(E_A^-;\alpha^-)$,$U(E_A^+;\alpha^+)$ and $L(L_A:S)$ are ideals of X $\forall\ t, s, \alpha^-\alpha^+ \in [0,1]$.

3.10 Theorem

An DEL-spherical fuzzy set A in X is a n-fold positive implicative BDEL-spherical fuzzy ideal of X if and only if the non-empty set $U(D_A;t)$,$L(E_A^-;\alpha^-)$,$U(E_A^+;\alpha^+)$ and $L(L_A:S)$ are n-fold positive implicative ideals of X $\forall\ t, s, \alpha^-\alpha^+ \in [0,1]$.

3.11 Theorem

Let A be a BDEL-spherical fuzzy ideal of X. Then A is n-fold implicative if and only if it satisfies the following condition:
$D_A((x*z)*(y*z) \geq D_A((x*y^{n+1})*z)$,
$E_A^-((x*z)*(y*z) \leq E_A^-((x*y^{n+1})*z)$,
$E_A^+((x*z)*(y*z) \geq E_A^+((x*y^{n+1})*z)$,
$L_A((x*z)*(y*z) \leq L_A((x*y^{n+1})*z)$, $\forall\ x, y, z \in X \longrightarrow (16)$
proof:
Assume that A is a n-fold positive implicative BDEl-spherical fuzzy ideal of X.Then A is a BDEL-spherical fuzzy ideal of X by prives Theorems and saties-fies the condition in equation(14)

$$\begin{aligned}((x*(y*z))*z)*z &= (((x*z)*(y*z))*z)\\ &\leq (x*y)*z \forall x,y,z \in X\end{aligned}$$

It follows by prives lemma

$$\begin{aligned}D_A((x*y^{n+1})*z) &\leq D_A(((x*(y*z))*z)*z)\\ E_A^-((x*y^{n+1})*z) &\geq E_A^-(((x*(y*z))*z)*z)\\ E_A^+((x*y^{n+1})*z) &\leq E_A^+(((x*(y*z))*z)*z)\\ L_A((x*y^{n+1})*z) &\geq L_A(((x*(y*z))*z)*z) \forall x,y,z \in X \longrightarrow (17)\end{aligned}$$

By using equation (3),(14)and (17),we have

$$\begin{aligned}D_A((x*z)*(y*z) &= D_A((x*(y*z)*z)\\ &\geq D_A(((x*(y*z))*z)*z)\\ &\geq D_A((x*y^{n+1})*z)\\ E_A^-((x*z)*(y*z) &= E_A^-((x*(y*z)*z)\\ &\leq E_A^-(((x*(y*z))*z)*z)\\ &\leq E_A^-((x*y^{n+1})*z)\\ E_A^+((x*z)*(y*z) &= E_A^+((x*(y*z)*z)\\ &\geq E_A^+(((x*(y*z))*z)*z)\\ &\geq E_A^+((x*y^{n+1})*z) \ and\\ L_A((x*z)*(y*z) &= L_A((x*(y*z)*z)\\ &\leq L_A(((x*(y*z))*z)*z)\\ &\leq L_A((x*y^{n+1})*z)\end{aligned}$$

Hence equation (16) is valid. Conversely,let A be a BDEL-spherical fuzzy ideal of X. which satisfies the condition equation (16). if we put $z = y$ in equation (16) and (1), then we obtain the condition (14).Therefore A is a n-fold positive implicative BDEL-spherical fuzzy ideal of X by prives thoerem.

3.12 Theorem

Let A be an DEL- spherical fuzzy set in X. then A is a n-fold positive implicative BDEL-spherical fuzzy ideal of X if and only if it satiesfies the condition Equation (9)(10) and
$D_A(x*y) \geq \min\ \{D_A(((x*y^{n+1})*y)*z), D_A(z)\}$
$E_A^-(x*y) \leq \max\ \{E_A^-(((x*y^{n+1})*y)*z), E_A^-(z)\}$
$E_A^+(x*y) \geq \min\ \{E_A^+(((x*y^{n+1})*y)*z), E_A^+(z)\}$
$L_A(x*y) \leq \max\ \{L_A(((x*y^{n+1})*y)*z), L_A(z)\}\forall\ x, y, z \in X \longrightarrow (18)$
proof:
Assume that A is a n-fold positive implicative BDEL-spherical fuzzy ideal of X. Then A is a BDEL- spherical fuzzy ideal of X and so on the conditions equation (9)and (10) are valid.Using (11),(1),(3)and (16),we have

$$\begin{aligned}
D_A(x*y) &\geq min\{D_A(((x*y^{n+1})*z), D_A(z)\} \\
&= min\{D_A(((x*z)*y^{n+1})*(y*y)), D_A(z)\} \\
&\geq min\{D_A(((x*z)*y^{n+1})*y), D_A(z)\} \\
&= min\{D_A(((x*y^{n+1})*y)*z), D_A(z)\} \\
E_A^-(x*y) &\leq max\{E_A^-(((x*y^{n+1})*z), E_A^-(z)\} \\
&= max\{E_A^-(((x*z)*y^{n+1})*(y*y)), E_A^-(z)\} \\
&\leq max\{E_A^-(((x*z)*y^{n+1})*y), E_A^-(z)\} \\
&= max\{E_A^-(((x*y^{n+1})*y)*z), E_A^-(z)\} \\
E_A^+(x*y) &\geq min\{E_A^+(((x*y^{n+1})*z), E_A^+(z)\} \\
&= min\{E_A^+(((x*z)*y^{n+1})*(y*y)), E_A^+(z)\} \\
&\geq min\{E_A^+(((x*z)*y^{n+1})*y), E_A^+(z)\} \\
&= min\{E_A^+(((x*y^{n+1})*y)*z), E_A^+(z)\} \\
\text{and } L_A(x*y) &\leq max\{L_A(((x*y^{n+1})*z), L_A(z)\} \\
&= max\{L_A(((x*z)*y^{n+1})*(y*y)), L_A(z)\} \\
&\leq max\{L_A(((x*z)*y^{n+1})*y), L_A(z)\} \\
&= max\{L_A(((x*y^{n+1})*y)*z), L_A(z)\}\ \forall x, y, z \in X
\end{aligned}$$

Conversly ,Let A be an DEL-spherical fuzzy set in X which satesfies condition equation (9),(10)and (18).Then

$$\begin{aligned}
D_A(x) &= D_A(x*0) \\
&\geq min\{D_A(((x*0^{n+1})*0)*z), D_A(z)\} \\
&= min\{D_A(x*z), D_A(z)\} \\
E_A^-(x) &= E_A^-(x*0) \\
&\leq max\{E_A^-(((x*0^{n+1})*0)*z), E_A^-(z)\} \\
&= max\{E_A^-(x*z), E_A^-(z)\} \\
E_A^+(x) &= E_A^+(x*0) \\
&\geq min\{E_A^+(((x*0^{n+1})*0)*z), E_A^+(z)\} \\
&= min\{E_A^+(x*z), E_A^+(z)\} \\
L_A(x) &= L_A(x*0) \\
&\leq max\{L_A(((x*0^{n+1})*0)*z), L_A(z)\} \\
&= max\{L_A(x*z), L_A(z)\} \forall x, y, z \in X.
\end{aligned}$$

Hence A is BDEL-spherical fuzzy ideal of X. Taking Z=0 in equation (18) and using (1)and (2) implyes that

$$\begin{aligned}
D_A(x*y) &\geq min\{D_A(((x*y^{n+1})*y)*0), D_A(0)\} \\
&= min\{D_A(((x*y^{n+1})*y), D_A(0)\} \\
&= \{D_A(((x*y^{n+1})*y)\} \\
E_A^-(x*y) &\leq max\{E_A^-(((x*y^{n+1})*y)*0), E_A^-(0)\} \\
&= max\{E_A^-((x*y^{n+1})*y), E_A^-(0)\} \\
&= \{E_A^-((x*y^{n+1})*y)\} \\
E_A^+(x*y) &\geq min\{E_A^+(((x*y^{n+1})*y)*0), E_A^+(0)\} \\
&= min\{E_A^+(((x*y^{n+1})*y), E_A^+(0)\} \\
&= \{E_A^+(((x*y^{n+1})*y)\} \quad \text{and}
\end{aligned}$$

$$\begin{aligned} L_A(x*y) &\leq max\{L_A(((x*y^{n+1})*y)*z), L_A(z)\} \\ &= max\{L_A(((x*y^{n+1})*y)*0), L_A(0)\} \\ &= \{L_A(((x*y^{n+1})*y)\} \forall x, y, z \in X. \end{aligned}$$

It follows from theorem (3.11) that A is a n-fold positive implicative BDEL-spherical fuzzy ideal of X.

3.13 Proposition

Every BDEL-spherical fuzzy ideal A of X satisfies the following assertion. $x * y \leq z \Rightarrow D_A(x*y) \geq \min\{D_A(y), D_A(z)\}$
$E_A^-(x) \leq \max\{E_A^-((y), E_A^-(z)\}$
$E_A^+(x) \geq \min\{E_A^+(y), E_A^+(z)\}$
$L_A(x) \leq \max\{L_A(y), L_A(z)\} \forall\ x, y, z \in X \longrightarrow (19)$
proof:
Let $x, y, z \in X$ be such that $x * y \leq z \Rightarrow$,then

$$\begin{aligned} D_A(x*y) &\geq min\{D_A(((x*y^{n+1})*z), D_A(z)\} \\ &= min\{D_A(0), D_A(z)\} \\ &= D_A(z). \\ E_A^-(x*y) &\leq max\{E_A^-(((x*y^{n+1})*z), E_A^-(z)\} \\ &= min\{E_A^-(0), E_A^-(z)\} \\ &= E_A^-(z). \\ E_A^+(x*y) &\geq min\{E_A^+(((x*y^{n+1})*z), E_A^+(z)\} \\ &= min\{E_A^+(0), E_A^+(z)\} \\ &= E_A^+(z). \text{ and} \\ L_A(x*y) &\leq max\{L_A(((x*y^{n+1})*z), L_A(z)\} \\ &= max\{L_A(0), L_A(z)\} \\ &= L_A(z). \end{aligned}$$

It follows that

$$\begin{aligned} D_A(x) &\geq min\{D_A(((x*y^{n+1})*z), D_A(z)\} \\ &\geq min\{D_A(y), D_A(z)\} \\ E_A^-(x) &\leq max\{E_A^-(((x*y^{n+1})*z), E_A^-(z)\} \\ &\leq max\{E_A^-(y), E_A^-(z)\} \\ E_A^+(x) &\geq min\{E_A^+(((x*y^{n+1})*z), E_A^+(z)\} \\ &\geq min\{E_A^+(y), E_A^+(z)\} \\ L_A(x) &\leq max\{L_A(((x*y^{n+1})*z), L_A(z)\} \\ &\leq max\{L_A(y), L_A(z)\} \end{aligned}$$

Hence the proof.

we provide condition for a DEL-spherical fuzzy set to be a BDEL-spherical fuzzy ideal in BCK/BCI-algebra.

3.14 Theorem

Every DEL-spherical fuzzy set in X satisfying equation (9),(10) and (19) is a BDEL-spherical fuzzy ideal of X.

Proof:

Let A be a DEL- spherical fuzzy set in X (9),(10) and (19).Note that $x*(x*y) \leq y \ \forall x, y \in X$. It follows that from equation (19)that

$$\begin{aligned} D_A(x) &\geq min\{D_A(x*y), D_A(y)\} \\ E_A^-(x) &\leq max\{E_A^-((x*y), E_A^-(y)\} \\ E_A^+(x) &\geq min\{E_A^+(x*y), E_A^+(y)\} \\ L_A(x) &\leq max\{L_A(x*y), L_A(y)\} \ \forall x, y \in X. \end{aligned}$$

Therefore A is a BDEL-spherical fuzzy ideal of X.

3.15 Theorem

An DEL-spherical fuzzy set Ain X is a BDEL-spherical fuzzy ideal of X if and only if (D_A, E^-) and (E^+, L) are intuitionistic fuzzy ideals of X.

proof:
straight forward.

3.16 Theorem

For any non-empty subset I of X.Let A be an DEL-spherical fuzzy set in X. If A is a n-fold positive implicative BDEL-spherical fuzzy set ideal of X.Then I is a n-fold positive implicative ideal of X.
Proof:
If A is a n-fold positive implicative BDEL-spherical fuzzy ideal of X, then A is a BDEL-spherical fuzzy ideal of X and satiefies equation (14) and (16). Let $x, y \in X$ be such that then $(x * y) * y \in I$

$$\begin{aligned}
D_A(x * y) &\geq D_A((x * y^{n+1}) * y) \\
&= t. \\
E_A^-(x * y) &\leq E_A^-((x * y^{n+1}) * y) \\
&= \alpha^- \\
E_A^+(x * y) &\geq E_A^+((x * y^{n+1}) * y) \\
&= \alpha^+ \\
L_A(x * y) &\leq L_A((x * y^{n+1}) * y) \\
&= S.
\end{aligned}$$

and so $x * y \in I$ therefore I is a n-fold positive implicative ideal of X.

3.17 Proposition

Every n-fold positive implicative BDEL-spherical fuzzy ideal A of X satisfies the following condition.
$(((x * y^{n+1)} * y) * a) * b = 0$ implies
$D_A(x * y) \geq \min \{D_A(a), D_A(b)\}$
$E_A^-(x * y) \leq \max \{E_A^-(a), E_A^-(b)\}$
$E_A^+(x * y) \geq \min \{E_A^+(a), E_A^+(b)\}$
$L_A(x * y) \leq \max \{L_A(a), L_A(b)\} \forall\ x, y, a, b \in X.$

Proof:

Assume that A is a n-fold positive implicative BDEL-spherical fuzzy ideal of X. Then A is a BDEL-spherical fuzzy ideal of X. Let $a, b, x, y \in X$ such that $(((x * y^{n+1}) * y) * a) * b = 0$, then

$$\begin{aligned} D_A(x*y) &\geq D_A((x*y^{n+1})*y) \\ &\geq min\{D_A(a), D_A(b)\} \\ E_A^-(x*y) &\leq E_A^-((x*y^{n+1})*y) \\ &\leq max\{E_A^-(a), E_A^-(b)\} \\ E_A^+(x*y) &\geq E^+((x*y^{n+1}*y) \\ &\geq min\{E_A^+(a), E_A^+(b)\} \\ &\text{and} \\ L_A(x*y) &\leq L_A((x*y^{n+1}*y) \\ &\leq max\{L_A(a), L_A(b)\}. \end{aligned}$$

by prives theorem and proposition .Hence (20) is valid.

3.18 Theorem

If an DEL-spherical fuzzy set A in X. satisfies the condition equation (9)and (20),then A is a n-fold positive implicative BDEL-spherical fuzzy ideal of X.

Proof:

Let A be an DEL-spherical fuzzy set in X which satiesfies the condition equation (9)and (20),It is clear that the condition equation (10)is included by the condition equation (20).Let $x, a, b \in X$ be such that $x * a \leq b$ then $(((x * 0^{n+1}) * 0) * a) * b = 0$, and so

$$\begin{aligned} D_A(x) &= D_A((x*0) \\ &\geq min\{D_A(a), D_A(b)\} \\ E_A^-(x) &= E_A^-(x*0) \\ &\leq max\{E_A^-(a), E_A^-(b)\} \end{aligned}$$

$$\begin{aligned} E_A^+(x) &= E^+(x*0) \\ &\geq min\{E_A^+(a), E_A^+(b)\} \text{ and} \\ L_A(x) &\leq L_A(x*0) \\ &\leq max\{L_A(a), L_A(b)\} \end{aligned}$$

By equation (1) and (20). Hence A is a BDEL-spherical fuzzy ideal of X by prives theorem.since
$(((x*y^{n+1})*y)*((x*y^{n+1})*y))*0 = 0 \; \forall \; x, y, \in X.$

$$\begin{aligned} \text{we have} D_A(x*y) &\geq min\{D_A((x*y^{n+1})*y), D_A(0)\} \\ &= D_A((x*y^{n+1})*y) \\ E_A^-(x*y) &\leq max\{E_A^-((x*y^{n+1})*y), E_A^-(0)\} \\ &= E_A^-((x*y^{n+1})*y) \\ E_A^+(x*y) &\geq min\{E_A^+((x*y^{n+1})*y), E_A^+(0)\} \\ &= E_A^+((x*y^{n+1})*y) \\ L_A(x*y) &\leq max\{L_A((x*y^{n+1})*y), L_A(0)\} \\ &= L_A((x*y^{n+1})*y). \end{aligned}$$

By equation (20) It follows from theorem (3.17) that A is a n-fold positive implicative DEL-spherical fuzzy ideal of X.

3.19 Conclusion

This paper presented a new concept on spherical fuzzy sets and its algebric structures together with ideal theory in BCK/BCI algebra. A DEL-spherical fuzzy set to be a n-fold positive implicative BDEL-spherical fuzzy ideal are provided. Characterization of n-fold positive implicative BDEL-spherical fuzzy ideals are displyed.

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